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MINIMUM THICKNESS OF A LIQUID FILM FLOWING VERTICALLY DOWN A SOLID SURFACE

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Abstract—The problem of rupture of a thin liquid film flowing down a vertical surface is considered. This rupture into rivulets is assumed to occur when both the continuous film and the rivulets carry the same mass flow and total (surface plus kinetic) energy and when, moreover, the latter exhibits in the rivulet configuration a local minimum. From the resulting theory, an earlier theory by Hobler can be retrieved as a special case. Calculations are performed for a gravity driven film and results are compared with several earlier theories.

NOMENCLATURE

e, specific energy per unit width;
E, energy;
$f(\theta_0)$, function defined by equation (18);
h, film thickness;
h_0^+, dimensionless film thickness;
$\dot{m}$, mass flow rate;
p_L, pressure in liquid phase;
P_v, pressure in vapor phase;
R, radius of rivulet surface;
w, velocity;
x, y, coordinates;
X, ratio of wetted to total surface.

Greek symbols

θ_0, contact angle;
λ, film width;
μ, viscosity;
ρ, density;
σ, surface tension;
$\psi(\theta_0)$, function defined by equation (20).

Subscripts

f, film;
riv, rivulet;
fs, liquid–solid interface;
fg, liquid–vapor interface;
sg, solid–vapor interface.

INTRODUCTION

THE CONDITIONS under which a thin liquid film, driven along a solid surface by gravity or by shear stresses applied at the "free" surface, breaks down into a series of rivulets, leaving the solid surface partially exposed, are of great importance in a number of technical applications. Among them are distillation and other direct-contact processes and equipment. The problems of liquid film breakdown are closely related to the dry patch formation on heated surfaces, which is of importance in safety studies of nuclear reactors cooled either by liquid metal or by water. The appearance of the boiling crisis in a flow situation is related to the conditions of maintenance of a continuous liquid film on the heated surface. Of more immediate interest to the present authors are the implications with respect to liquid flow on the surfaces of stator blades of the last few stages of very large steam turbines. Here, the existence of a continuous film or its breakdown into rivulets is of central importance in the problem of liquid removal from the surface by evaporation. Such removal, by using hot steam on the inside of hollow stator blades, would minimize the danger of large droplets forming from the film near the trailing edge of the stator blade. This in turn would reduce subsequent erosion damage to the rotor blades [1–3]. The problem of evaporation of such a film involves estimating conditions for the breakdown of the film into rivulets.

The problem of stability of a liquid film has been the subject of many analytical investigations based on the classical linear stability theory [4–10].* These investigations yield conditions for the growth of small disturbances in the film and on its surface. They cannot be expected to provide information concerning the actual conditions under which rivulets appear or the detailed mechanism involved in the film breakdown. This is due to at least two factors, the first being that when a disturbance grows sufficiently to present a real danger of film breakdown, it is very unlikely that a linear theory will continue to offer an adequate description. The second, and more important difficulty, is that a very important parameter of the physical phenomenon, namely the contact angle does not enter the theory at all.

Another approach to the problem of breakdown of thin films and subsequent rewetting, or stability of the dry patch, has been offered initially by Hartley and Murgatroyd [11] who investigated the equilibrium of forces acting at the stagnation point of a film, which point also constitutes the beginning of a dry patch in the film. They also offered an alternate criterion based on the assumption that a stable film configuration corresponds to a minimum power transmission by the film in the form of kinetic and surface energy. This

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*This listing is not intended to be complete but rather to include the work of several independent groups.

alternate criterion again suffers from the drawback that it does not involve the contact angle and, further, the basic assumption appears quite arbitrary. Both analyses are concerned with the stability of the broken film configuration. They yield the limiting thickness of an unbroken film, from which the stable rivulet configuration results.

Comparison of the force criterion with experiment [12] indicated qualitative agreement but required the use of unrealistic contact angles in the theory if satisfactory quantitative agreement were to be achieved. Subsequent developments of the Hartley and Murgatroyd theory [13–15] were aimed at the effects of shear and form-drag forces, non-uniform temperature distribution on the film surface, and the added pressure caused by the thrust of evaporating molecules.

An approach somewhat similar to the minimum power criterion of [12] but with a firmer theoretical basis was adopted by Hobler [16]. He considered the total (kinetic plus surface) energy contained in a given streamwise length of broken film. If in this configuration the total energy exhibits a local minimum Hobler concludes that the film will break. If the broken configuration exhibits no energy minimum the conclusion is that the continuous film is stable. The theory has been compared with experiments [17, 18] and has been extended to films exposed to centrifugal force fields [19]. The theory does account for the effect of the contact angle but offers no information concerning the geometry or spacing of the rivulets resulting from the film break-up.

More recently Bankoff [20] formulated a criterion for the formation of rivulets in the shape of segments of a circle. He assumed equality of the mass flow and total energy in the film and rivulet configuration and further assumed effectively that rivulets can form adjacent to each other with no intervening dry surface. It is not difficult to show that these conditions cannot be simultaneously satisfied and therefore one would expect his theory not to yield any minimum stable film thickness values. The fact that the author did obtain numerical results, albeit inexplicably low ones, is due to a numerical error in his equation (15). If the error is removed imaginary minimum film thicknesses result from Bankoff's theory, as might be expected.

The present work offers yet another approach to the problem enlarging on the ideas of both Hobler and Bankoff. Following Bankoff we consider the film to break up into rivulets in the shape of segments of a circle, consistent with a uniform surface tension. It is assumed, however, that the rivulet has a base narrower than the uniform unbroken film thus allowing a dry space between adjoining rivulets. The ratio X of the rivulet base, to the corresponding undisturbed film width represents a basic variable in the Hobler approach. Further following Bankoff, the energy in the unbroken film and the broken-film configuration are assumed equal. Finally, the energy of the broken-film configurations is required to have a minimum for $X < 1$, as proposed by Hobler, if the configuration is to be stable.

In this manner it is possible to calculate not only the minimum thickness of the stable film, but also the radius of curvature and initial spacing of the rivulets resulting from the break-up of the film.

ANALYSIS

We consider a homogeneous liquid film of uniform thickness h flowing down a vertical plane. The flow is assumed to be fully developed and laminar with the velocity given by $w(y)$. The mechanical energy of the film consists of the kinetic energy of the fluid in the film together with the surface energies associated with the solid–liquid interface and the "free" surface. Per unit length of film streamwise and for a width λ of the film this energy is given by

$$E_f = \left[\int_0^h \frac{\rho}{2} w^2(y)\,dy + \sigma_{sf} + \sigma_{fg}\right]\lambda \tag{1}$$

and per unit width of the film

$$e_f = \frac{E_f}{\lambda} = \int_0^h \frac{\rho}{2} w^2(y)\,dy + \sigma_{sf} + \sigma_{fg}. \tag{2}$$

Similarly the mass flow per unit width of the film is given by

$$\frac{\dot{m}_f}{\lambda} = \int_0^h \rho w(y)\,dy. \tag{3}$$

Let us now consider the same flow in the form of rivulets. The radius of curvature of the rivulet surface is determined by the equation

$$P_L - P_v = \frac{\sigma}{R}. \tag{4}$$

On a vertical surface P_L does not vary across the rivulet and P_v is also constant. If σ is also considered constant, which is consistent with uniform temperature, then R is constant and the cross-section of the rivulet forms a segment of a circle, subtending at the center an angle of $2\theta_0$, where θ_0 is the contact angle. Let us consider next the cross-section of the rivulet divided into narrow strips of width dx and height $h(x)$. The velocity profile in such a strip is assumed to be the same as the profile in a uniform film of the same thickness $h(x)$.

The mass flow per unit width of the original film is given by

$$\frac{\dot{m}_{riv}}{\lambda} = \frac{2}{\lambda}\int_0^{R\sin\theta_0}\int_0^{h(x)} \rho w(x,y)\,dx\,dy. \tag{5}$$

When the film breaks, the mass flow intensities given by equations (3) and (5) are equal which upon introduction of

$$h(x) = R(\cos\theta - \cos\theta_0) \quad 0 \leqslant x \leqslant R\sin\theta_0, \quad 0 \leqslant \theta \leqslant \theta_0 \tag{6}$$

yields a relationship between the rivulet radius R and the critical film thickness h_0. The total energy of a rivulet and associated dry surface may be written

$$E_{riv} = \left[2\int_0^{R\sin\theta_0}\int_0^{h(x)} \frac{\rho}{2} w^2(x,y)\,dy\,dx + 2R\sin\theta_0\sigma_{fs} + 2R\theta_0\sigma_{fg} + (\lambda - 2R\sin\theta_0)\sigma_{sg}\right]. \tag{7}$$

From the equilibrium of the surface tension forces at the point of contact of the three phases

$$\sigma_{sg} = \sigma_{fs} + \sigma_{fg}\cos\theta_0 \tag{8}$$

so that per unit width λ of the original film

$$e_{\mathrm{riv}} = \frac{E_{\mathrm{riv}}}{\lambda} = \frac{\rho}{\lambda}\int_0^{R\sin\theta_0}\int_0^{h(x)} w^2(x,y)\,\mathrm{d}x\,\mathrm{d}y + \left(\frac{2R\theta_0}{\lambda} + \cos\theta_0 - \frac{R\sin 2\theta_0}{\lambda}\right)\sigma_{fg} + \sigma_{fs}. \tag{9}$$

Introducing the ratio of the surface wetted by the rivulets to the total surface

$$X = \frac{2R\sin\theta_0}{\lambda} \tag{10}$$

e_{riv} becomes a function of X.

The rivulet is stable if its total energy e_{riv} is a minimum at $X = X_0$

$$\frac{\partial e_{\mathrm{riv}}}{\partial X} = 0; \qquad \frac{\partial^2 e_{\mathrm{riv}}}{\partial X^2} > 0 \tag{11}$$

while at the same time

$$e_f = e_{\mathrm{riv}}; \qquad X_0 \leqslant 1. \tag{12}$$

Equations (11) and (12) determine the minimum critical film thickness and the corresponding value of X, while the equality of mass flow yields the rivulet radius R and, finally, the spacing λ. In the subsequent section the above method is applied to a laminar free-falling film.

THE LAMINAR GRAVITY-DRIVEN FILM ON A VERTICAL WALL

The velocity profile appropriate to this case is

$$w(y) = \frac{g\rho}{\mu}\left(\frac{y^2}{2} - yh\right) \tag{13}$$

so that

$$e_f = \int_0^h \frac{1}{2}\frac{\rho^3 g^2}{\mu^2}\left(\frac{y^2}{2} - yh\right)^2 \mathrm{d}y + \sigma_{fg} + \sigma_{fs} = \frac{1}{15}\frac{\rho^3 g^2}{\mu^2}h^5 + \sigma_{fg} + \sigma_{fs} \tag{14}$$

and

$$\frac{\dot{m}_f}{\lambda} = \int_0^h \frac{\rho^2 g}{\mu}\left(\frac{y^2}{2} - yh\right)\mathrm{d}y = \frac{\rho^2 g}{\mu}\frac{h^3}{3}. \tag{15}$$

For the rivulet

$$\dot{m}_{\mathrm{riv}} = 2\int_0^{R\sin\theta_0}\int_0^{h(x)} \rho w(x,y)\,\mathrm{d}x\,\mathrm{d}y = \frac{2}{3}\frac{\rho g}{\mu}R^4\int_0^{\theta_0}(\cos\theta - \cos\theta_0)^3\cos\theta\,\mathrm{d}\theta = \frac{2}{3}\frac{\rho^2 g}{\mu}R^4 f(\theta_0) \tag{16}$$

and

$$\frac{\dot{m}_{\mathrm{riv}}}{\lambda} = \frac{2}{3}\frac{\rho^2 g}{\mu}R^3\frac{R\sin\theta_0}{\lambda}\frac{f(\theta_0)}{\sin\theta_0} = \frac{\rho^2 g}{\mu}X\frac{R^3}{3}\frac{f(\theta_0)}{\sin\theta_0} \tag{17}$$

where

$$f(\theta_0) = \int_0^{\theta_0}(\cos\theta - \cos\theta_0)^3\cos\theta\,\mathrm{d}\theta = -\tfrac{1}{4}\cos^3\theta_0\sin\theta_0 - \tfrac{13}{8}\cos\theta_0\sin\theta_0 - \tfrac{3}{2}\theta_0\sin^2\theta_0 + \tfrac{15}{8}\theta_0. \tag{18}$$

For the total energy of the rivulet and associated dry surface one obtains from equations (7) and (8)

$$E_{\mathrm{riv}} = 2\int_0^{R\sin\theta_0}\int_0^{h(x)}\frac{\rho}{2}\frac{g^2\rho^2}{\mu^2}\left[\frac{y^2}{2} - yh(x)\right]^2\mathrm{d}y\,\mathrm{d}x + (2R\theta_0 + \lambda\cos\theta_0 - 2R\sin\theta_0\cos\theta_0)\sigma_{fg} + \lambda\sigma_{fs}$$
$$= \frac{2}{15}\frac{\rho^3 g^2}{\mu^2}R^6\int_0^{\theta_0}(\cos\theta - \cos\theta_0)^5\cos\theta\,\mathrm{d}\theta + (2R\theta_0 + \lambda\cos\theta_0 - 2R\sin\theta_0\cos\theta_0)\sigma_{fg} + \lambda\sigma_{fs}$$
$$= \frac{2}{15}\frac{\rho^3 g^2}{\mu^2}R^6\psi(\theta_0) + (2R\theta_0 + \lambda\cos\theta_0 - 2R\sin\theta_0\cos\theta_0)\sigma_{fg} + \lambda\sigma_{fs} \tag{19}$$

where

$$\psi(\theta_0) = \int_0^{\theta}(\cos\theta - \cos\theta_0)^5\cos\theta\,\mathrm{d}\theta = \theta_0\left(\tfrac{5}{16} + \tfrac{15}{4}\cos^2\theta_0 + \tfrac{5}{2}\cos^4\theta_0\right) - \sin\theta_0\left(\tfrac{113}{48}\cos\theta_0 + \tfrac{97}{24}\cos^3\theta_0 + \tfrac{1}{6}\cos^5\theta_0\right).^{*} \tag{20}$$

For the mean energy per unit width there results

$$e_{\mathrm{riv}} = \frac{E_{\mathrm{riv}}}{\lambda} = \frac{1}{15}\frac{\rho^3 g^2}{\mu^2}XR^5\frac{\psi(\theta_0)}{\sin\theta_0} + X\sigma_{fg}\left(\frac{\theta_0}{\sin\theta_0} - \cos\theta_0\right) + \sigma_{fg}\cos\theta_0 + \sigma_{fs}. \tag{21}$$

Equating the mass flow per unit width of flow, given by equations (15) and (17) one obtains for the critical film condition

$$\left(\frac{h_0}{R}\right)^3 = X\frac{f(\theta_0)}{\sin\theta_0} \tag{22}$$

which upon substitution into equation (21) yields

$$e_{\mathrm{riv}} = \frac{\rho^3 g^2}{15\mu^2}X^{-2/3}\left[\frac{\sin\theta_0}{f(\theta_0)}\right]^{5/3}h_0^5\frac{\psi(\theta_0)}{\sin\theta_0} + X\sigma_{fg}\left(\frac{\theta_0}{\sin\theta_0} - \cos\theta_0\right) + \sigma_{fg}\cos\theta_0 + \sigma_{fs}. \tag{23}$$

If the rivulet configuration is to be stable the above energy per unit width should exhibit a minimum at $X = X_0 < 1$. Thus differentiating equation (23) with respect to X, equating to zero and solving for X_0

$$X_0 = \left[\frac{2}{45}\frac{\rho^3 g^2}{\mu^2\sigma_{fg}}\frac{\psi(\theta_0)}{\sin\theta_0}\left(\frac{\theta_0}{\sin\theta_0} - \cos\theta_0\right)^{-1}\right]^{3/5}\frac{\sin\theta_0}{f(\theta_0)}h_0^3 = \left[\frac{2}{3}\frac{\psi(\theta_0)}{\sin\theta_0}\left(\frac{\theta_0}{\sin\theta_0} - \cos\theta_0\right)^{-1}\right]^{3/5}\frac{\sin\theta_0}{f(\theta_0)}h_0^{+3} \tag{24}$$

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*Equations for $f(\theta_0)$ and $\psi(\theta_0)$ were given by Bankoff [20], with an unfortunate arithmetic error in the latter. The coefficient of $\cos^3\theta_0$ was given as $\frac{49}{8}$. This leads to further erroneous results referred to earlier in the paper.

where the dimensionless critical film thickness h_0^+ is defined by

$$h_0^+ = \left(\frac{\rho^3 g^2}{15\mu^2\sigma_{fg}}\right)^{1/5} h_0. \quad (25)$$

Since also the total energy of the continuous film and the rivulet configuration must be the same, comparison of equations (14) and (23) yields upon substitution of equation (24)

$$\frac{\rho^3 g^2}{15\mu^2\sigma_{fg}} h_0^5 + (1-\cos\theta_0) = \left(\frac{\rho^3 g^2}{15\mu^2\sigma_{fg}}\right)^{3/5} h_0^3 \times \left(\frac{2}{3}\right)^{3/5}\left(\frac{5}{2}\right) \times \left[\frac{\sin\theta_0}{f(\theta_0)}\right]\left[\frac{\psi(\theta_0)}{\sin\theta_0}\right]^{3/5}\left(\frac{\theta_0}{\sin\theta_0} - \cos\theta_0\right)^{2/5} \quad (26)$$

or simply

$$h_0^{+5} + (1-\cos\theta_0) - G(\theta_0) h_0^{+3} = 0. \quad (26a)$$

The dimensionless critical film thickness h_0^+ determined by the above equation and the corresponding values of X_0, as calculated from equation (24), are shown as functions of the contact angle θ_0 in Fig. 1.

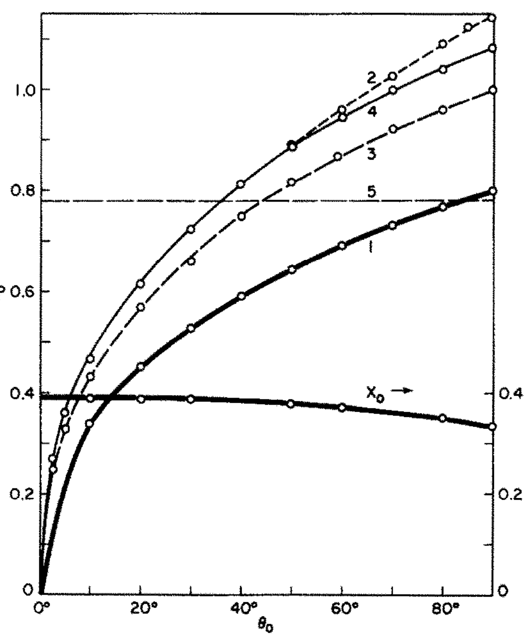

FIG. 1. Comparison of several film breakdown theories: (1) present theory; (2) equation (27); (3) equation (29), Murgatroyd force criterion; (4) equation (31), Hobler; (5) equation (30), Murgatroyd power criterion; X_0, equation (24).

Further calculations can be performed for specific fluids giving the critical film thickness h_0, the radius of the rivulets R from equation (22) and finally the spacing of the rivulets λ from equation (10).

It might be noted here that in the Hobler theory [16] it is assumed that a film will break whenever a stable rivulet can exist for $X < 1$. Here X is defined as the ratio of wetted-to-total surface area, without specific assumptions regarding the shape of the cross-section of the rivulets. The critical film thickness is taken as corresponding to $X = 1$. Thus from equation (24) there results

$$h_0^+ = \left[\frac{3}{2}\frac{\sin\theta_0}{\psi(\theta_0)}\left(\frac{\theta_0}{\sin\theta} - \cos\theta_0\right)\right]^{1/5}\left[\frac{f(\theta_0)}{\sin\theta_0}\right]^{1/3}. \quad (27)$$

These results are also plotted in Fig. 1.

Finally, if the result of comparing equations (14) and (23) is left in terms of X and X is set equal to 1, the Bankoff theory [20] results:

$$h_0^{+5} = \frac{\theta_0 - \sin\theta_0}{\sin\theta_0 - \left[\frac{f(\theta_0)}{\sin\theta_0}\right]^{-5/3}\psi(\theta_0)}. \quad (28)$$

With the correct form of $\psi(\theta_0)$ this yields no real positive values of h_0^+ for any value of θ_0 between 0 and 90°. For comparison purposes we show also in Fig. 1 the results of Hartley and Murgatroyd [11], which when recast in the terminology of the present paper take the form

$$\text{Force Criterion} \quad h_0^+ = (1-\cos\theta_0)^{1/5} \quad (29)$$

$$\text{Power Criterion} \quad h_0^+ = 0.779. \quad (30)$$

The original theory of Hobler [16] may be expressed in the form

$$h_0^+ = (\tfrac{3}{2})^{1/5}(1-\cos\theta_0)^{1/5} \quad (31)$$

which differs from equation (27) in that no account is taken of the rivulet shape.

COMPARISON WITH EXPERIMENTAL DATA

Comparison of the present theory with available experimental data on minimum film thickness tends to be somewhat inconclusive. On the one hand there are the data of Hobler *et al.* [18]. To the author's knowledge, they are the only ones including measured contact angles. The comparison is reproduced below:

System	θ_0	h^+ Experimental	Hobler theory	Present theory
Water–aluminum	37.7	0.728	0.793	0.550
Water–glass	35.8	0.787	0.777	0.540
Water–copper	53.0	0.901	0.902	0.660
Water–stainless steel	36.3	0.900	0.781	0.545
Water–varnish	56.8	0.977	0.925	0.680

On the other hand there are the data of Norman and McIntyre [21]* and of Simon and Hsu [22]. Direct comparison here is not possible since neither reference includes the contact angle measurements. It is possible, however, to deduce the contact angle required in the theory in order to reproduce the experimental data.

*Reference [21] has been brought to the author's attention by one of the reviewers. This help is gratefully acknowledged.

Thus:

System	$h_{0,\mathrm{exp}}$ (μm)	h_0^+	θ_0 Hobler theory	θ_0 Present theory
Water–smooth copper 30°C [21]	136	0.365	6°	16°
Water–smooth copper 45°C [21]	104	0.314	5°	14°
Water–smooth copper 60°C [21]	67	0.224	3°	10°
Water–smooth copper 75°C [21]	70	0.255	4°	11°
Water–chromium plate 30°C [21]	137	0.367	6°	16°
Water–chromium plate 45°C [21]	115	0.344	6°	15°
Water–chromium plate 60°C [21]	97	0.324	5°	14°
Water–chromium plate 75°C [21]	80	0.292	5°	13°
Water–glass 27°C [22]	147	0.387	6°	19°

Thus it may be concluded that the present theory predictions are uniformly too low by approximately a factor of 1.5 with regard to the experimental results of Hobler but show reasonable agreement with the results of references [21] and [22]. In those latter cases, use of more realistic values of θ_0 would lead to generally too high predictions. This apparent inconsistency points up the need for more systematic experimental data.

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MINIMUM D'EPAISSEUR D'UN FILM LIQUIDE S'ECOULANT SUR UNE SURFACE VERTICALE

Résumé—On considère le problème de la rupture d'un mince film liquide s'écoulant sur une surface verticale. On suppose que le ruissellement se produit lorsque le film continu et les ruisselets issus de sa division transportent la même masse et la même énergie totale (surfacique et cinétique) et que, de plus, cette dernière présente dans la configuration du ruissellement un minimum local. La théorie à laquelle on aboutit permet de retrouver comme cas particulier une théorie dûe à Hobler. Les calculs sont effectués pour un film liquide soumis à des forces de pesanteur et les résultats sont comparés à ceux de plusieurs théories antérieures.

MINIMALE DICKE EINES AUF EINER FESTEN OBERFLÄCHE VERTIKAL ABLAUFENDEN FLÜSSIGKEITSFILMS

Zusammenfassung—Es wird das Problem des Aufbrechens eines an einer vertikalen Oberfläche ablaufenden dünnen Flüssigkeitsfilms diskutiert. Dabei wird davon ausgegangen, daß eine Bachbildung dann eintritt, wenn sowohl der kontinuierliche Film wie die einzelnen Rinnsale denselben Massenstrom und dieselbe Gesamtenergie (Oberflächenenergie + kinetische Energie) aufweisen und wenn außerdem die Gesamtenergie im Falle der Bachströmung ein lokales Minimum aufweist. Aus der heiraus gewonnenen Theorie läßt sich eine frühere Theorie von Hobler als Spezialfall ableiten. Es werden Berechnungen für schwerkraftkontrollierte Filme durchgeführt und die Ergebnisse mit mehreren früheren Theorien verglichen.

МИНИМАЛЬНАЯ ТОЛЩИНА ПЛЕНКИ ЖИДКОСТИ, СТЕКАЮЩЕЙ ПО ВЕРТИКАЛЬНОЙ ТВЕРДОЙ ПОВЕРХНОСТИ

Аннотация — Рассматривается задача о разрыве тонкой пленки жидкости, стекающей по вертикальной поверхности. Предполагается, что при разрыве пленка и ручейки сохраняют одинаковые массовые расходы и полные потоки энергии (поверхностная плюс кинетическая). Кроме того, ручейковая геометрия характеризуется локальным минимумом этой энергии. На основании полученной теории более раннюю теорию Хоблера можно рассматривать как частный случай. Выполнены расчеты для пленки жидкости, стекающей под действием силы тяжести. Результаты сравниваются с некоторыми ранее полученными данными.